Integration by Substitution

Example 1.

$\int -t^3\sqrt{3t^4+4}\,dt$

Let $u = 3t^4+4$, $du = 12\underline{t^3\,dt}$

$t^3\,dt = \frac{du}{12}$

$\int -t^3\sqrt{3t^4+4}\,dt = -\frac{1}{12}\int \sqrt{u}\,du = \frac{-1}{12}\int u^{\frac{1}{2}}du$

$= \frac{-1}{12}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = -\frac{(3t^4+4)^{\frac{3}{2}}}{18} + C$

Example 2.

$\int \frac{-5(\ln y)^4}{y}\,dy$

Let $u = \ln y$, $du = \frac{1}{y}\,dy$

$\int \frac{-5(\ln y)^4}{y}\,dy = \int -5(u)^4\cdot du$

$= -5\int u^4\,du = -5\frac{u^5}{5} + C = -u^5 + C$

$= -[\ln y]^5 + C$

Example 3.

$\int 6xe^{-4x^2}dx$ Let $u = 4x^2$, $du = 8x\,dx$.

$=$ $x\,dx = \frac{du}{8}$

$$\int 6e^{-u}\frac{du}{8} = \frac{3}{2}\int e^{-u}du = -\frac{3}{2}e^{-u} + C$$

$$= -\frac{3}{2}e^{-4x^2} + C$$

Example 4.

$\int_2^3 10xe^{\left(x^2+2\right)}dx$ Let $u = x^2+2$. Then $du = 2x\,dx$.

$x\,dx = \frac{du}{2}$.

when $x=2$, $u=6$. when $x=3$, $u=11$

$$\int_2^3 10xe^{(x^2+2)}dx = \int_6^{11} 10 \cdot e^u \cdot \frac{du}{2}$$

$$= 5\int_6^{11} e^u du = 5e^u\Big|_6^{11} = 5(e^{11} - e^6)$$